

Solving Trigonometric Equations with Identities

For the following exercises, find all solutions exactly that exist on the interval $[0, 2\pi)$.

1. $\csc^2 t = 3$

2. $\cos^2 x = \frac{1}{4}$

3. $2 \sin \theta = -1$

4. $\tan x \sin x + \sin(-x) = 0$

5. $9 \sin \omega - 2 = 4 \sin^2 \omega$

6. $1 - 2 \tan(\omega) = \tan^2(\omega)$

For the following exercises, use basic identities to simplify the expression.

7. $\sec x \cos x + \cos x - \frac{1}{\sec x}$

8. $\sin^3 x + \cos^2 x \sin x$

For the following exercises, determine if the given identities are equivalent.

9. $\sin^2 x + \sec^2 x - 1 = \frac{(1 - \cos^2 x)(1 + \cos^2 x)}{\cos^2 x}$

10. $\tan^3 x \csc^2 x \cot^2 x \cos x \sin x = 1$

Sum and Difference Identities

For the following exercises, find the exact value.

11. $\tan\left(\frac{7\pi}{12}\right)$

12. $\cos\left(\frac{25\pi}{12}\right)$

13. $\sin(70^\circ)\cos(25^\circ) - \cos(70^\circ)\sin(25^\circ)$

14. $\cos(83^\circ)\cos(23^\circ) + \sin(83^\circ)\sin(23^\circ)$

For the following exercises, prove the identity.

15. $\cos(4x) - \cos(3x)\cos x = \sin^2 x - 4\cos^2 x \sin^2 x$ 16. $\cos(3x) - \cos^3 x = -\cos x \sin^2 x - \sin x \sin(2x)$

For the following exercise, simplify the expression.

17.
$$\frac{\tan\left(\frac{1}{2}x\right) + \tan\left(\frac{1}{8}x\right)}{1 - \tan\left(\frac{1}{8}x\right)\tan\left(\frac{1}{2}x\right)}$$

For the following exercises, find the exact value.

18. $\cos\left(\sin^{-1}(0) - \cos^{-1}\left(\frac{1}{2}\right)\right)$ 19. $\tan\left(\sin^{-1}(0) + \sin^{-1}\left(\frac{1}{2}\right)\right)$

Double-Angle, Half-Angle, and Reduction Formulas

For the following exercises, find the exact value.

20. Find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ given $\cos \theta = -\frac{1}{3}$ and θ is in the interval $[\frac{\pi}{2}, \pi]$.

21. Find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ given $\sec \theta = -\frac{5}{3}$ and θ is in the interval $[\frac{\pi}{2}, \pi]$.

22. $\sin\left(\frac{7\pi}{8}\right)$

23. $\sec\left(\frac{3\pi}{8}\right)$

For the following exercises, use [Figure 1](#) to find the desired quantities.

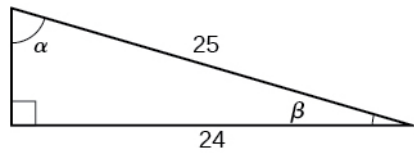


Figure 1

24. $\sin(2\beta)$, $\cos(2\beta)$, $\tan(2\beta)$, $\sin(2\alpha)$, $\cos(2\alpha)$, and $\tan(2\alpha)$

25. $\sin\left(\frac{\beta}{2}\right)$, $\cos\left(\frac{\beta}{2}\right)$, $\tan\left(\frac{\beta}{2}\right)$, $\sin\left(\frac{\alpha}{2}\right)$, $\cos\left(\frac{\alpha}{2}\right)$, and $\tan\left(\frac{\alpha}{2}\right)$

For the following exercises, prove the identity.

26. $\frac{2\cos(2x)}{\sin(2x)} = \cot x - \tan x$ 27. $\cot x \cos(2x) = -\sin(2x) + \cot x$

For the following exercises, rewrite the expression with no powers.

28. $\cos^2 x \sin^4(2x)$ 29. $\tan^2 x \sin^3 x$